

## ROCKING INTENSITY MEASURES: FROM INTERFACE VARIABLES TO RESPONSE PROXIES

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**Abstract:** *In the context of the performance-based earthquake engineering (PBEE) framework an intensity measure (IM) is the interface (or interfacing) variable that links the seismic hazard with the structural fragility/vulnerability for the risk assessment of structures. On the other hand, from the standpoint of structural dynamics, an IM may be used as a proxy for predicting the structural response under a specific ground motion. Hence, depending on the usage per case, different criteria of optimality should be employed. An interface variable needs to be efficient (low conditional dispersion) and sufficient (low dependence on seismological parameters), whereas also its hazard needs to be assessable via available ground motion prediction equations. For the case of a proxy, hazard computability is not necessary, whereas the most important criterion is the capability of the IM to predict the engineering demand parameter (EDP) within a (simple) regression model. Thus, a response proxy needs mainly to offer high correlation and low fitting errors within IM-EDP regression models. Herein, after addressing these two different cases of IM usage, a comparison of alternative IMs for rocking structures is presented, mainly focusing on their use within a PBEE framework for risk assessment. Simple rocking bodies are employed for running incremental dynamic analysis with a set of 105 ordinary (no-pulse-like, no-long-duration) natural ground motions. It is shown that some well-established IMs are both efficient and sufficient for the case of rocking bodies. Still, due to the nature of rocking response, some (e.g., peak ground acceleration) tend to be optimal only in specific regions of response (e.g., rocking initiation). Moreover, dependence on the magnitude of the earthquake is found to be higher than for the distance from the rupture. Finally, IMs that are inefficient and insufficient for risk assessment can be at the same time very effective when used as response proxies.*

### Introduction

The rocking oscillator has attracted considerable research interest by virtue of being widely applicable as the seismic response mechanism of different types of structures, ranging from monuments (e.g., Psycharis *et al.* 2013) to modern resilient structures (e.g., Manzo and Vassiliou 2021). As a result, many studies investigate the rocking response of rigid bodies (rocking dynamics) under natural (mainly pulse-type) ground motions or even single pulse excitation (e.g., Housner 1963; Yim *et al.* 1980; Ishiyama 1982; Makris and Konstantinidis 2003; Dimitrakopoulos and DeJong 2012; Makris and Vassiliou 2013). Rocking is a negative stiffness problem and thus very sensitive to the ground motion time-history waveform, the geometry of the block and the impacts of the block to its base during rocking motion (Yim *et al.* 1980). Hence, recent studies (e.g., Kazantzi *et al.* 2021; Lachanas and Vamvatsikos 2022, Lachanas *et al.* 2022; Kazantzi *et al.* 2022; Lachanas *et al.* 2023a; Lachanas *et al.* 2023b) have adopted the probabilistic treatment of the problem based on the principles of performance-based earthquake engineering (PBEE) aiming to offer a complete and solid framework for the standardization of the seismic response of rigid rocking bodies for application in risk or vulnerability studies.

In modern vulnerability studies that are performed following the PBEE paradigm (Cornell *et al.* 2002) for assessing the seismic risk/loss/damage of engineering structures, the intensity measure (IM) has the role of the interface variable (IM-V) that links the seismic hazard with the structural response (Cornell *et al.* 2002; Kazantzi and Vamvatsikos 2015). Typically, it refers to a scalar (or vector) quantity for which the seismic hazard for a specific site can be calculated, whereas its selection has a strong influence on the risk estimates (Kohrangi *et al.* 2016a; Kohrangi *et al.* 2016b). Still, when working outside the PBEE framework, an IM can be employed as a response

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proxy (IM-P) for predicting the structural response under specific ground motions. In this case the seismic hazard calculation is not a requirement and thus an IM-P can have a more complex form.

Up till now, studies in the field of earthquake engineering have focused on finding optimal IMs for conventional engineering structures (e.g., Luco and Cornell 2007; Padgett *et al.* 2008; Kazantzi and Vamvatsikos 2015). Regarding rocking, studies that were mainly focused on the rocking dynamics and the effect of ground motion characteristics on the rocking response, sometimes proposed some robust rocking IMs; still without providing strong evidence for their usage within PBEE (e.g., Makris and Black 2004; Psycharis *et al.*, 2013; Dimitrakopoulos and Paraskeva 2015). On the other hand, recent studies have focused on testing alternative IMs for structures of rocking behavior via statistical testing (e.g., Shokrabadi and Burton 2017; Kavvadias *et al.* 2017; Giouvanidis and Dimitrakopoulos 2018; Sieber *et al.* 2022). However, with the exception of Shokrabadi and Burton (2017), where the ability of assessing the seismic hazard was employed as a rigorous criterion for the IMs examined therein, in the aforementioned studies, no distinction between IM-Vs and IM-Ps was employed, whereas their findings are useful mainly for the IM-P usage. Hence, herein, after clarifying the two distinct roles an IM can play in earthquake engineering, a comparative study of alternative IMs is presented focusing on their usage as IM-Vs within the PBEE framework aiming to provide optimal IM-Vs for vulnerability studies of rocking structures.

### Criteria for selection - Interface variables versus response proxies

For the conventional usage of an IM as an IM-V three basic criteria are required in order to eliminate the bias and the computational effort for the risk assessment (Kazantzi and Vamvatsikos 2015). The first criterion refers to the ability of assessing the seismic hazard (*hazard computability*) or in other words the existence of appropriate ground motion prediction equations (GMPEs) for the IM at hand. The second criterion is *efficiency*, which refers to low record-to-record dispersion of the structural response when running response history analysis under suites of ground motions. This dispersion regards either the engineering demand parameter (EDP) values for given levels of the IM or the IM values for given EDP values. The third criterion is *sufficiency*, which refers to the dependency of the IM on the seismological parameters (e.g., magnitude, distance from the rupture) of the ground motion. An IM-V is considered as sufficient when the structural response remains unaffected by the seismological characteristics of the ground motions employed, meaning that the structural demand calculations are not altered significantly when ground motions of different seismological features are employed. In addition to these three main criteria for the selection, there is an extra desired requirement for an IM-V. Typically, in PBEE the performance assessment is made via response history analysis by scaling sets of ground motions, e.g., via incremental dynamic analysis (IDA, Vamvatsikos and Cornell 2002). Hence, a robust IM-V needs to be scalable, meaning that the application of a scale factor to the components of an individual ground motions should lead to an equal scaling of the IM-V, implying linear scalability.

On the other hand, for the usage as an IM-P the aforementioned criteria are not strict requirements, since an IM-P is employed outside the realm of PBEE. The main requirement for a robust IM-P is the ability for predicting the EDP and visa-versa. This is achieved by an overall good fitting in the IM-EDP space, at least when adopting a power law fitting model (linear model in log-log space) under sets of ground motions of different intensities (Cornell *et al.* 2002). Hence, the crucial parameter for selecting an IM-P is the coefficient of determination ( $R^2$ , Weisberg 2005), with a high  $R^2$  indicating the effectiveness of fitting and of the examined IM-P consequently.

### Modeling, analysis and examined intensity measures

#### *Modeling and Analysis*

The typical model of the planar rectangular rigid rocking block standing freely on a rigid base and subjected to horizontal excitation (Figure 1a, Housner 1963) was employed for the investigation. According to this pure rocking model of a rectangular block with base width of  $2b$  and total height of  $2h$ , the parameters that govern the seismic response are: the slenderness angle  $\alpha = \tan^{-1}(b/h)$ , the half diagonal  $R = \sqrt{b^2 + h^2}$  from which the characteristic frequency  $p = \sqrt{(3g)/(4R)}$  is also calculated, and the coefficient of restitution  $\eta$ , which is associated with the energy loss due to the impacts of the block to its base during rocking. Herein, the proposed  $\alpha$ -

dependent approximation of Housner (1963) for the latter was employed (i.e.,  $\eta = 1 - (3/2)(\sin \alpha)^2$ ).

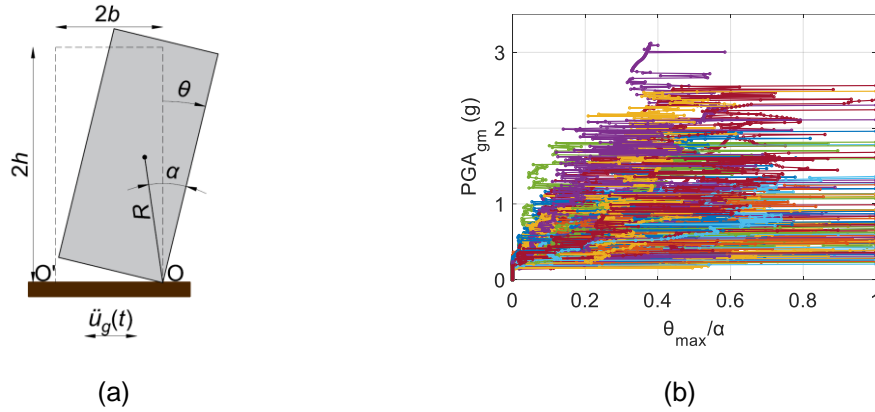


Figure 1. (a) Planar rocking block on a rigid base. (b) IDA results for block B2 under 105 ordinary ground motions using  $PGA_{gm}$  as IM.

The geometric and dynamic characteristics of the blocks that are employed herein are presented in Table 1. Of the selected blocks, B2 resembles a simplified planar model-analogue of a monolithic ancient column of the Temple of Aphaia in Aegina, Greece (Lachanas and Vamvatsikos 2022), whereas the other two blocks have been selected to resemble a taller and slender rocking block (B1) versus a shorter and less slender one (B3).

Block	2b (m)	2h (m)	$\alpha$ (rad)	R (m)	p (s <sup>-1</sup> )	$\eta$
B1	1.50	15.00	0.0997	7.54	0.9880	0.99
B2	0.95	5.29	0.1777	2.69	1.6546	0.95
B3	1.00	4.00	0.2450	2.06	1.8892	0.91

Table 1. Geometric and dynamic characteristics of the rocking blocks under investigation.

IDA was employed for assessing the seismic response of the three blocks of Table 1 by using a large set of 105 ordinary (no-pulse-like, no-long-duration) ground motions (Lachanas and Vamvatsikos 2022). The numerical analysis was performed by using the scripts of Vassiliou (2021) for solving the rocking equation of motion under horizontal excitation. The proposed by Lachanas and Vamvatsikos (2022) approach for running IDA to rocking oscillators was adopted. Specifically, one horizontal (arbitrary) component per ground motion was assigned to the model and the scaling was made by using a constant step of 0.01 g for the peak ground acceleration (PGA). As EDP, the peak absolute rocking angle  $\theta_{max}$  normalized by the slenderness angle,  $\tilde{\theta} = \theta_{max}/\alpha$ , was employed. Analysis was terminated at the first PGA level where overturning of the block ( $\tilde{\theta} \geq 1.00$ ) was observed without taking into consideration the possible resurrections of the block; this approach is considered as acceptable for practical purposes (Lachanas and Vamvatsikos 2022). After analysis, the  $PGA - \tilde{\theta}$  results were converted into alternative IMs by refitting IDA curves to the desired IM per case. Figure 1b presents the IDA results for block B2 with the  $PGA - \tilde{\theta}$  results converted into  $PGA_{gm} - \tilde{\theta}$ , where  $PGA_{gm}$  is the geometric mean of the peak ground acceleration of the two horizontal components of the ground motion. The IM values that are produced per modern GMPEs do not refer to a specific direction but practically correspond to the geomean IM values (Baker and Cornell 2006). Thus, geomean IMs (e.g.,  $PGA_{gm}$ ) are preferred IM-Vs within the PBEE framework in order to avoid introducing some undesired bias when linking hazard (geomean IM values) with structural analysis.

#### Examined intensity measures

Table 2 presents the IMs that are compared within the present study. The first nine IMs (No. 1–9) refer to single/averaged spectral ordinates that have been used in vulnerability studies for conventional structures. These are considered from the beginning as IM-Vs since there are GMPEs available for them in the literature. Hence, for their definition the geomean component (subscript gm) is employed. The first two IM-Vs examined are  $PGA_{gm}$  and the peak ground velocity  $PGV_{gm}$ . Both these IMs are commonly used in rocking studies, with the former being associated directly with rocking uplift ( $PGA \approx g \tan \alpha$ ) while the latter has been proposed as a

more robust rocking IM (e.g., Makris and Black 2004; Psycharis *et al.* 2013). The next three examined IM-Vs refer to the 5% damped first-mode spectral acceleration,  $S_{agm}(T_1, 5\%)$ , which is one of the most commonly-used IMs for buildings. Since the rocking oscillator does not have a constant oscillation frequency (Housner 1963; Makris and Konstantinidis 2003), three different periods of 0.5, 1.0, 2.0 s are employed to examine  $S_{agm}(T_1, 5\%)$  in different areas of the elastic spectra. The rest of the examined IM-Vs (No. 6–9) refer to different period-range cases of  $AvgS_{agm}$ .  $AvgS_{agm}$  is the geometric mean of the spectral acceleration ordinates calculated from the two horizontal components over a period range and has been proposed as efficient and sufficient IM for the case of conventional engineering structures (e.g., Kazantzi and Vamvatsikos 2015). Four different period ranges with a constant step of 0.1 s are assumed in order to test  $AvgS_{agm}$  cases with narrow period ranges (i.e.,  $AvgS_{agm1}$ ) against others with broader ones (i.e.,  $AvgS_{agm4}$ ).

The remaining two IMs (No. 10-11) refer to rocking-specific IMs that have been recently proposed by Giouvanidis and Dimitrakopoulos (2018). The first one is the uniform duration,  $t_{uni}$ , which is calculated as the total duration of the record where the ground acceleration ( $\ddot{u}_g$ ) exceeds the rocking initiation threshold ( $|\ddot{u}_g| > g \tan \alpha$ ). The second is the cumulative absolute velocity estimated over the same duration for the exceeding values of ground acceleration ( $CAV_{exc}$ ). Both are considered as IM-Ps since they cannot be linked with seismic hazard, at the time, and cannot be used as IM-Vs. Additionally, they do not scale linearly during IDA with  $t_{uni}$ , inevitably being saturated and limited by the record duration. However, the nine IM-Vs plus the two IM-Ps are tested equally within this paper.

No.	ID (units)	Definition	No.	ID (units)	Definition
1	$PGA_{gm}$ (g)		7	$AvgS_{agm2}$ (g)	$AvgS_{agm}(0.3-3.0s)$
2	$PGV_{gm}$ (cm/s)		8	$AvgS_{agm3}$ (g)	$AvgS_{agm}(0.5-4.0s)$
3	$S_{agm1}$ (g)	$S_{agm}(0.5s, 5\%)$	9	$AvgS_{agm4}$ (g)	$AvgS_{agm}(0.1-4.0s)$
4	$S_{agm2}$ (g)	$S_{agm}(1.0s, 5\%)$	10	$t_{uni}$ (s)	
5	$S_{agm3}$ (g)	$S_{agm}(2.0s, 5\%)$	11	$CAV_{exc}$ (cm/s)	
6	$AvgS_{agm1}$ (g)	$AvgS_{agm}(0.1-1.5s)$			

Table 2. Alternative competing IM-Vs (No. 1–9) and IM-Ps (No. 10–11).

## Comparing the alternative IMs

### Testing efficiency and sufficiency of the IM-Vs

Efficiency is associated with low record-by-record variability and is tested herein by employing the methodology proposed by Vamvatsikos and Cornell (2005). Specifically, record-by-record dispersion per IM ( $\beta_{IM}$ ) is calculated on an IM given EDP basis (IM|EDP), by dividing the full  $\tilde{\theta}$ -range  $[0, 1]$  into discrete  $\tilde{\theta}$  levels and then calculating the dispersion as the standard deviation of the natural logarithm of the 105 IM values for any given  $\tilde{\theta}$  level. The main advantage of this IM|EDP approach is that it does not only give a general view for global efficiency of an IM but it also offers results regarding the localized efficiency at different stages of the structural response and thus the corresponding fragility curves. However, in order to calculate  $\beta_{IM}$  a functional inversion is needed for the case of rocking IDAs (Lachanas and Vamvatsikos 2022). As presented in Figure 1b, rocking IDAs show highly weaving behaviour. As a result, while a given IM level always corresponds to a single EDP value on a single IDA curve, when looking at vertical stripes (i.e., given EDP levels) multiple IM values on the same IDA curve may be found. Thus, an approximate functional inversion is needed in order to work on an IM|EDP basis. Herein, the low-bias median point inversion technique as proposed by Lachanas and Vamvatsikos (2022) is employed. According to it, for any given  $\tilde{\theta}$ -level, the median of the possible IM level intersections along the vertical on a single IDA curve is taken to construct the inverted IDA curve [i.e.,  $IM = f^{-1}(EDP)$ ].

Figure 3 presents the resulting  $\beta_{IM}$  of the nine examined IM-Vs for the three blocks under investigation. As illustrated, for all block cases there is no uniformly optimal IM at least among those examined herein.  $PGA_{gm}$  seems to be highly efficient in the rocking uplift neighbourhood (i.e.,  $\tilde{\theta} < 0.10$ ), showing dispersion less than 0.20, whereas it becomes inefficient for the higher range of rocking response up to overturning with  $\beta_{IM}$  values even exceeding 0.60. This could be considered as an expected finding since uplift of the block is directly associated with the ground

acceleration ( $|\ddot{u}_g| > g \tan \alpha$ ). Regarding  $S_{agm}$ , different periods, lead to different trends of  $\beta_{IM}$ , with lower periods approximating the behaviour of  $PGA_{gm}$  (e.g.,  $S_{agm1}$ ). For higher periods,  $S_{agm}$  seems to be inefficient for low  $\tilde{\theta}$  levels but showing at the same time efficiency for higher ones and close to overturning. Thus, a combination of spectral ordinates via  $AvgS_{agm}$  seems to be a more efficient choice, still having the inherent disadvantage of selecting a proper period range. As shown, selecting a narrow range of short-to-medium periods for  $AvgS_{agm1}$  leads to an IM-V that is efficient mainly close to rocking uplift. On the contrary, extensive period ranges with the inclusion of short-to-high periods like  $AvgS_{agm2}$  and  $AvgS_{agm4}$  or medium-to-high for  $AvgS_{agm3}$  can be optimal choices for  $\tilde{\theta} > 0.20$ . Finally,  $PGV_{gm}$  is found to be the most consistent rocking IM-V in the full range of rocking response showing uniformly  $\beta_{IM}$  values less than 0.50. Hence, even if it is considered as imperfect when compared to  $PGA_{gm}$  in the uplift area, or a variant of  $AvgS_{agm}$  close to overturning, it can be proposed as the only uniformly efficient IM-V for rocking vulnerability studies.

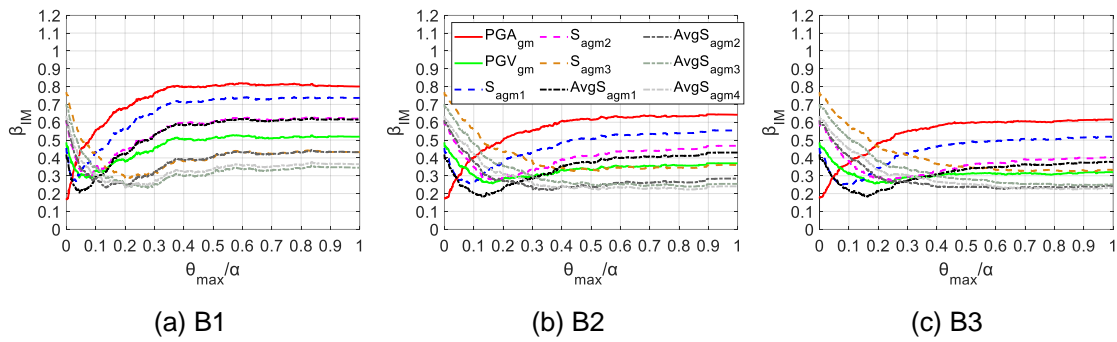


Figure 2. Efficiency test results for the nine IM-Vs for the three blocks under investigation.

Then, we move to the sufficiency test for the nine IM-Vs. Sufficiency refers to the ability of an IM-V to show low dependence on the seismological parameters of the ground motions. Herein, it is tested against the moment magnitude,  $M_w$ , and the Joyner-Boore distance from the rupture,  $R_{JB}$ . The 105 ground motions employed have  $M_w$  of 6.24–7.62 and distances  $R_{JB}$  of 0–69.95 km. Again, the sufficiency test is applied to the IM|EDP statistics by employing the proposed by Kazantzi and Vamvatsikos (2015) sufficiency test. Hence, the explained dispersion ( $\beta_{expl}$ ) by  $M_w$  and  $R_{JB}$  are calculated for given  $\tilde{\theta}$  levels as the square root of the square of the total record-to-record dispersion ( $\beta_{IM}^2$ ) minus the square of the unexplained dispersion ( $\beta_{unexpl}^2$ ).  $\beta_{unexpl}$  is calculated as the standard deviation of the regression residuals after applying a linear regression model to the 105  $\ln IM$  capacity values per each  $\tilde{\theta}$  level against  $M_w$  and  $R_{JB}$ .

Figure 3 presents the  $\beta_{expl}$  by  $M_w$  (a–c) and  $R_{JB}$  (d–f) for the nine IM-Vs and the three blocks under investigation. Regarding  $M_w$ , results follow in general those of the efficiency test of Figure 2. It is important here to note that similarly to the efficiency test there is no pre-defined limit of  $\beta_{expl}$  to define an IM-V as sufficient; still, in both cases, dispersion values as low as possible are desired.  $PGA_{gm}$  is the optimal sufficient IM-V against magnitude for rocking uplift, while it shows higher explained-by-magnitude dispersion than the rest of the candidates for the higher range of rocking response.  $AvgS_{agm3}$  and  $AvgS_{agm4}$  present low dispersions for  $\tilde{\theta} > 0.20$ , whereas  $PGV_{gm}$  shows a stable behaviour with low-to-medium dispersion in the entire range of response for all the examined block cases. Regarding  $R_{JB}$  results of  $\beta_{expl}$ , they offer stronger evidence of sufficiency of the nine IM-Vs. In all cases it does not exceed 0.15 indicating high sufficiency of the tested IM-Vs against  $R_{JB}$ . As a final note, results of Figure 3 indicate that there is somewhat higher dependence on the magnitude for the examined IM-Vs, where there is practically no dependence on the distance from the rupture.

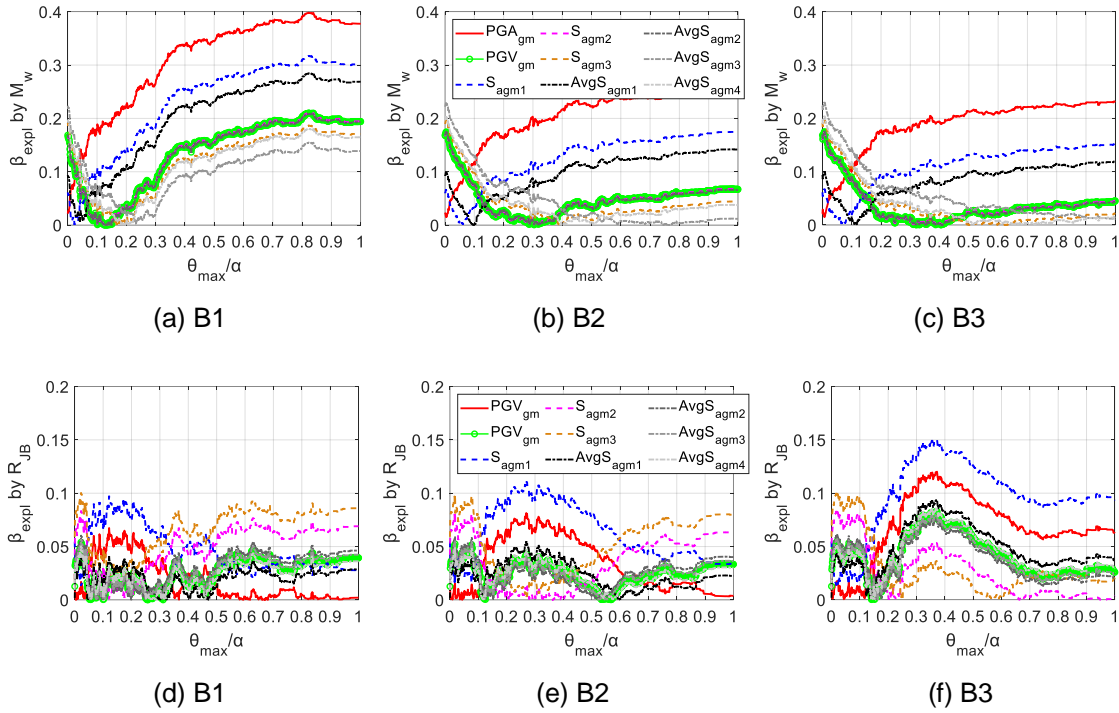


Figure 3. Explained dispersion by  $M_w$  (a–c) and  $R_{JB}$  (d–f) for the nine competing IM-Vs and the three blocks under investigation.

#### Testing the efficiency and sufficiency of the IM-Ps

Now, the tests of efficiency and sufficiency of the previous sections are employed for the two rocking-specific IM-Ps of Table 2. Figure 4 presents the  $\beta_{IM}$  and the  $\beta_{expl}$  by  $M_w$   $R_{JB}$  for blocks B1 and B3 of Table 1. As shown, both  $t_{uni}$  and  $CAV_{exc}$  are inefficient as interface variables in rocking vulnerability studies showing high record-to-record dispersion that exceeds 1.0 for the former or even 2.0 for the latter. These findings can be associated with the nature of these two IM-Ps that were constructed to be used with unscaled rather than scaled ground motions. The results of the sufficiency test are better. In general, like the IM-Vs of the previous section, there is higher dependency on  $M_w$  than on  $R_{JB}$ .  $\beta_{expl}$  of  $t_{uni}$  against  $M_w$  is about 0.20 for both B1 and B3, staying nearly stable in the full range of response. This dispersion is higher than that of most of the IM-Vs of Figure 3 but it should be associated with the considerably higher  $\beta_{IM}$  for  $t_{uni}$ . For  $CAV_{exc}$ ,  $\beta_{expl}$  reaches 0.50–0.60 but still it is a third or almost a quarter of the total dispersion. Overall, in addition to the absence of available GMPEs for them, the two examined IM-Ps are inefficient and moderately sufficient in comparison with the IM-Vs of the previous section. Still, they can be very effective response proxies when used as response proxies within linear regression models under suites of unscaled ground motions (Giouvanidis and Dimitrakopoulos 2018). The same findings also stand for block B2 but they are not shown herein for brevity.

## Conclusions

There are fundamental differences between the usage of an IM as an interface variable for vulnerability studies and its usage as a response proxy for predicting the structural response. Hence, the selection of an IM needs to be associated with the aims and the goals of a study, and then to employ the relative tests per usage in order to select an optimal IM. Regarding the usage as an IM-V and after testing for efficiency,  $PGA_{gm}$  is the obvious winner in the rocking uplift neighbourhood (i.e.,  $\tilde{\theta} < 0.05$ ), whereas it is inefficient for the rest of the range of rocking response up to overturning.  $AvgS_{agm}$  can be an optimal choice with the exception of the rocking uplift neighbourhood, whereas it has the disadvantage of the selection of a proper period range that may need to be adjusted per block case in order to produce an optimal IM-V.  $PGV_{gm}$  shows more uniform efficiency even if it is less optimal than  $PGA_{gm}$  in the rocking uplift neighbourhood or a variant of  $AvgS_{agm}$  close to overturning. Regarding sufficiency, the dependence on magnitude follows the findings for efficiency, whereas all the candidates examined as potential

IM-Vs are found as sufficient against the distance from the rupture. Finally, some rocking-specific IMs are found to be unsuitable as interface variables; yet they can be employed robustly as IM-Ps. A further and more detailed investigation of the subject is offered by Lachanas *et al.* (2023b).

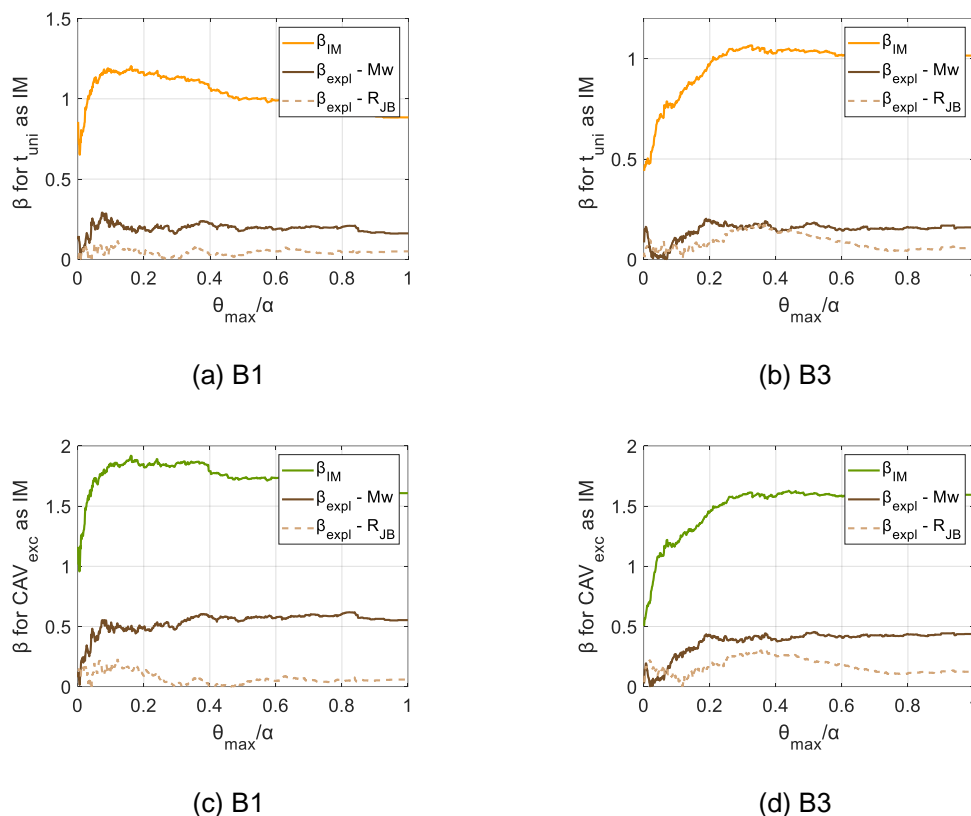


Figure 4. Efficiency and sufficiency tests of  $t_{uni}$  (a-b) and  $CAV_{exc}$  (c-d) for the blocks B1 and B3.

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### References

Baker JW and Cornell CA (2006), Which spectral acceleration are you using?, *Earthquake Spectra*, 22(2): 293–312 <https://doi.org/10.1193/1.2191540>

Cornell CA, Jalayer F, Hamburger RO and Foutch DA (2002), The probabilistic basis for the 2000 SAC/FEMA steel moment frame guidelines, *ASCE Journal of Structural Engineering*, 128(4): 526–533 [https://doi.org/10.1061/\(ASCE\)0733-9445\(2002\)128:4\(526\)](https://doi.org/10.1061/(ASCE)0733-9445(2002)128:4(526))

Dimitrakopoulos EG and DeJong MJ (2012), Revisiting the rocking block: closed-form solutions and similarity laws, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 468(2144): 2294–2318 <https://doi.org/10.1098/rspa.2012.0026>

Dimitrakopoulos EG and Paraskeva TS (2015), Dimensionless fragility curves for rocking response to near-fault excitations, *Earthquake Engineering and Structural Dynamics*, 44(12): 2015–2033 <https://doi.org/10.1002/eqe.2571>

Giouvanidis AI and Dimitrakopoulos EG (2018), Rocking amplification and strong-motion duration, *Earthquake Engineering and Structural Dynamics*, 47(10): 2094–2116. <https://doi.org/10.1002/eqe.3058>

- Housner GW (1963), The behavior of inverted pendulum structures during earthquakes, *Bulletin of the Seismological Society of America*, 53(2): 403–417
- Ishiyama Y (1982), Motions of rigid bodies and criteria for overturning by earthquake excitations, *Earthquake Engineering and Structural Dynamics*, 10(5): 635–650  
<https://doi.org/10.1002/eqe.4290100502>
- Kavvadias IE, Papachatzakis GA, Bantilas KE, Vasiliadis LK and Elenas A (2017), Rocking spectrum intensity measures for seismic assessment of rocking rigid blocks, *Soil Dynamics and Earthquake Engineering*, 101: 116–124 <https://doi.org/10.1016/j.soildyn.2017.07.021>
- Kazantzi AK and Vamvatsikos D (2015), Intensity measure selection for vulnerability studies of building classes, *Earthquake Engineering & Structural Dynamics*, 44(15): 2677–2694  
<https://doi.org/10.1002/eqe.2603>
- Kazantzi AK, Lachanas CG and Vamvatsikos D (2021), Seismic response distribution expressions for on-ground rigid rocking blocks under ordinary ground motions, *Earthquake Engineering and Structural Dynamics*, 50(12): 3311–3331 <https://doi.org/10.1002/eqe.3511>
- Kazantzi AK, Lachanas CG, and Vamvatsikos D (2022), Seismic response distribution expressions for rocking building contents under ordinary ground motions, *Bulletin of Earthquake Engineering*, 20: 6659–6682 <https://doi.org/10.1007/s10518-022-01424-w>
- Kohrangi M, Bazzurro P and Vamvatsikos D (2016a), Vector and scalar IMs in structural response estimation: Part I - Hazard Analysis, *Earthquake Spectra*, 32(3): 1507–1524  
<http://dx.doi.org/10.1193/053115EQS080M>
- Kohrangi M, Bazzurro P and Vamvatsikos D (2016b), Vector and scalar IMs in structural response estimation: Part II - Building Demand Assessment, *Earthquake Spectra*, 32(3): 1525–1543 <http://dx.doi.org/10.1193/053115EQS081M>
- Lachanas CG and Vamvatsikos D (2022), Rocking incremental dynamic analysis, *Earthquake Engineering and Structural Dynamics*, 51(3): 688–703 <https://doi.org/10.1002/eqe.3586>
- Lachanas CG, Vamvatsikos D and Vassiliou MF (2022), The influence of the vertical component of ground motion on the probabilistic treatment of the rocking response of free-standing blocks, *Earthquake Engineering and Structural Dynamics*, 51(8): 1874–1894  
<https://doi.org/10.1002/eqe.3643>
- Lachanas CG, Vamvatsikos D and Dimitrakopoulos EG (2023a) Statistical property parameterization of simple rocking block response, *Earthquake Engineering and Structural Dynamics*, 52(2): 394–414 <https://doi.org/10.1002/eqe.3765>
- Lachanas CG, Vamvatsikos D and Dimitrakopoulos EG (2023b), Intensity measures as interfacing variables versus response proxies: The case of rigid rocking blocks, *Earthquake Engineering and Structural Dynamics*, 52(6): 1722–1739 <https://doi.org/10.1002/eqe.3838>
- Luco N and Cornell CA (2007), Structure-specific scalar intensity measures for near-source and ordinary earthquake ground motions, *Earthquake Spectra*, 23(2): 357–392  
<http://dx.doi.org/10.1193/1.2723158>
- Makris N and Konstantinidis D (2003), The rocking spectrum and the limitations of practical design methodologies, *Earthquake Engineering and Structural Dynamics*, 32(2): 265–289  
<https://doi.org/10.1002/eqe.223>
- Makris N and Black CJ (2004), Evaluation of peak ground velocity as a “good” intensity measure for near-source ground motions, *Journal of Engineering Mechanics (ASCE)*, 130(9): 1032–1044 [https://doi.org/10.1061/\(ASCE\)0733-9399\(2004\)130:9\(1032\)](https://doi.org/10.1061/(ASCE)0733-9399(2004)130:9(1032))
- Makris N and Vassiliou MF (2013), Planar rocking response and stability analysis of an array of free-standing columns capped with a freely supported rigid beam, *Earthquake Engineering and Structural Dynamics*, 42(3): 431–449 <https://doi.org/10.1002/eqe.2222>
- Manzo NR and Vassiliou MF (2021), Cyclic tests of a precast restrained rocking system for sustainable and resilient seismic design of bridges, *Engineering Structures*, 113620  
<https://doi.org/10.1016/j.engstruct.2021.113620>
- Padgett JE, Nielson BG and DesRoches R (2008), Selection of optimal intensity measures in probabilistic seismic demand models of highway bridge portfolios, *Earthquake Engineering and Structural Dynamics*, 37(5): 711–725 <https://doi.org/10.1002/eqe.782>
- Psycharis IN, Fragiadakis M and Stefanou I (2013), Seismic reliability assessment of classical columns subjected to near-fault ground motions, *Earthquake Engineering and Structural Dynamics*, 42(14): 2061–2079 <https://doi.org/10.1002/eqe.2312>



- Shokrabadi M and Burton HV (2017), Ground Motion Intensity Measures for Rocking Building Systems, *Earthquake Spectra*, 33(4): 1533–1554 <https://doi.org/10.1193/040816eqs055m>
- Sieber M, Vassiliou MF and Anastasopoulos I (2022), Intensity measures, fragility analysis and dimensionality reduction of rocking under far-field ground motions, *Earthquake Engineering and Structural Dynamics*, 51(15): 3639–3657 <https://doi.org/10.1002/eqe.3740>
- Vamvatsikos D and Cornell CA (2005), Developing efficient scalar and vector intensity measures for IDA capacity estimation by incorporating elastic spectral shape information, *Earthquake Engineering and Structural Dynamics*, 34(13): 1573–1600 <https://doi.org/10.1002/eqe.496>
- Vamvatsikos D and Cornell CA (2002), Incremental dynamic analysis, *Earthquake Engineering and Structural Dynamics*, 31(3): 491–514 <https://doi.org/10.1002/eqe.141>
- Vassiliou MF (2021). Script for the seismic response of a planar rocking block, MATLAB script: available at <http://hdl.handle.net/20.500.11850/521016>, <http://dx.doi.org/10.3929/ethz-b-000521016> (Accessed 27/03/2023)
- Yim C-S, Chopra AK and Penzien J (1980), Rocking response of rigid blocks to earthquakes, *Earthquake Engineering and Structural Dynamics*, 8(6): 565–587 <https://doi.org/10.1002/eqe.4290080606>
- Weisberg S (2005), *Applied linear regression*, John Wiley & Sons. DOI:10.1002/0471704091